parameters used in the code. The time breakdown is shown in Table 1. With the present procedure a mesh of 10^5 cells requires about 191 s of CPU time. It can be seen that the most expensive part of the algorithm is the elimination of nodes of degree higher than seven. The cause of this inefficiency has not been tracked in detail but must be attributed to a bad practice in the implementation of the algorithm. If the node-degree homogenization techniques are not employed, then the time is reduced to 70 s approximately.

It has been reported[2] that the time necessary to generate a two-dimensional grid with 10^5 elements it is about 17 s on an SGI Indigo[2] R4400. The same author claims that the equivalent method for the generation of a surface grid requires three times as much CPUtime as the two-dimensional planar procedure. Thus the total cost can be estimated in about 1 min of CPU time, which is more or less the same amount obtained if the node-degree homogenization technique is removed from Table 1.

Conclusions

A method for the efficient generation of unstructured grids on arbitrary surfaces based in Steiner triangulations has been presented. The mesh-generation system uses an internal representation of the surface to isolate the grid-generation and CAD systems. The algorithm does not need the tracking of fronts, and therefore its implementation is attractive because of its simplicity. A special filter has been devised to avoid the generation of sawtooth patterns on the surface during the early stages of the gridding process, and this has proved to be an essential part of the method. The quality of the surface and planar grids obtained by the present approach are similar, but still below the attainable by means of an AF point-placement strategy.

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Continuous-Phase Properties of Homogeneous Particle-Laden Turbulent Flows

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Nomenclature

C_D	=	particle drag coefficient
D	=	dissipation factor; Eq. (2)
d_p	=	particle diameter
f_i	=	volume fraction of region i
n''	=	particle number flux
U_p	=	mean streamwise relative velocity of a particle
u, v	=	instantaneous streamwise and cross-stream gas velocity
$\bar{u}, \bar{v}$	=	mean streamwise and cross-stream gas velocity
$\bar{u}', \bar{v}'$	=	rms fluctuating streamwise and cross-stream velocity
ε	=	local rate of dissipation of turbulence kinetic energy
φ	=	average generic property of the overall flow
φ_i	=	average generic property of region i of the flow

Subscripts

i	=	turbulent interwake region
w	=	particle wake region

Introduction

TURBULENCE generation is defined as the direct disturbance of the continuous-phase velocity field by the wakes of dispersed-phase objects in dispersed multiphase flows. Turbulence generation supplements the conventional production of turbulence caused by mean velocity gradients in the continuous phase; it is most important when dispersed-phase objects have large relative velocities (large Reynolds numbers) and relatively large relaxation times compared to characteristic turbulence times. Such conditions are typical of many practical dispersed multiphase flows having significant separated-flow effects, e.g., sprays, particle-laden jets, bubbly jets, rainstorms, etc. Motivated by these observations, the present investigation sought to develop methods to predict the overall continuous-phase properties of homogeneous particle-laden turbulent flows dominated by turbulence generation and to evaluate the predictions based on earlier measurements from this laboratory.[1–5]

Initial observations of turbulence generation in this laboratory considered uniform fluxes of nearly monodisperse spherical particles in still water and air to yield homogeneous and stationary flows where turbulence production was caused entirely by turbulence generation.[1,2] The local rate of dissipation of turbulence kinetic energy ε mainly controls continuous-phase properties in these flows and can be found as the local rate of loss of particle mechanical energy per unit volume, as follows:

$$\varepsilon = \pi n'' d_p^2 C_D U_p^2 / 8 \qquad (1)$$

All other properties of these flows, however, are not known and must be related to dissipation rates and particle properties. Finally,

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whereas simplified stochastic theories developed during these studies yielded encouraging correlations of turbulence intensities in terms of a dimensionless dissipation factor other properties were not predicted very well because of limited understanding about the particle wakes in these flows and their turbulent surroundings.[1,2]

Subsequent studies considered the properties of particle wakes at intermediate Reynolds numbers in turbulent environments typical of conditions for flows dominated by turbulence generation.[6,7] These wakes scaled similar to self-preserving laminar wakes but clearly were turbulent and mixed faster than laminar wakes as a result of the presence of turbulence in both the wakes and their surroundings; therefore, they were called laminarlike turbulent wakes. Recent observations of flows dominated by turbulence generation in upflowing air confirmed the presence of laminarlike turbulent wakes that were surrounded by a relatively large turbulent interwake region.[3–5] The turbulent interwake regions were found to be analogous to grid-generated isotropic turbulence in the final decay period, which are also widely recognized as being turbulent flows since the time of early observations of this flow by Batchelor and Townsend[8] more than 50 years ago, and yielded correlations of their properties [moments, probability density function (PDFs), etc.] in terms of the dimensionless dissipation factor and known properties of the dispersed phase.[4]

Motivated by these observations, the objective of the present investigation was to develop a way to predict the overall properties of the continuous phase of flows caused by turbulence generation, seeking to account for the random arrival of particle wakes and incorporating the known properties of laminarlike turbulent wakes and the turbulent interwake region. Information of this type would be measured in the continuous phase of flow dominated by turbulence generation and is of considerable importance as a result. The study was limited to homogeneous particle-laden turbulent flows involving uniform fluxes of nearly monodisperse spherical particles so that predictions could be evaluated based on the measurements of Refs. 1–3. The present description of the research is brief. (See Refs. 1–5 for additional details about the measurements and tabulations of data.)

Theoretical Methods

The present flows are assumed to consist of uniform fluxes of monodisperse spherical particles moving with known relative velocities (nearly equal to terminal velocities) within a homogeneous (nearly isotropic) and stationary turbulent continuous phase. A laminarlike turbulent wake is associated with each particle. The properties of the particle wakes must be considered to be part of the continuous-phase turbulence field because the arrival of particle wakes is random. Particle volume fractions were less than 0.003%, and their corresponding wake volume fractions were less than 2–30% so that effects of wake/wake interactions are small.[3] Then given particle properties, particle fluxes, and particle relative velocities, the dissipation factor of the flow D can be found from its definition, as follows[4]:

$$D = \frac{\varepsilon d_p (C_D/8)^{\frac{1}{2}}}{\left[\pi U_p^2 (U_p - \bar{u})\right]} \tag{2}$$

as discussed in earlier work.[3–5] (See Ref. 4 for discussion of the relationship between D and the properties of the turbulent interwake region.) Then given D and the particle properties, the results of Chen and Faeth[4] provide the properties of the turbulent interwake region. Given this information and the known values of particle Reynolds numbers (having values of 1.06, 3.73, and 9.90×10^2 for the present test particles having nominal diameters of 0.5, 1.1, and 2.2 mm, respectively), the mean and fluctuating properties of the particle wakes can be obtained from the results of Wu and Faeth[6,7] for laminarlike turbulent wakes in a straightforward manner.

Given appropriate average properties of the laminarlike turbulent wakes and the turbulent interwake region, the corresponding overall property of the continuous phase was estimated using conditional averages, as follows:

$$\phi = f_w \phi_w + f_i \phi_i \tag{3}$$

When computing the wake volume fraction, the small volume of the particle was ignored. Thus, the wake volumes included the region extending in the radial direction from the wake axis to the position where the mean streamwise velocity defect of the wake was $\exp(-2)$ of its value at the axis and for a streamwise distance extending to the point where the mean streamwise velocity defect at the axis was equal to the ambient r.m.s. velocity fluctuation level. Notably, doubling the size of this region had a negligible effect on the computed overall properties of the continuous phase.

Properties that will be considered in the following include $\phi = \bar{u}'^2$, $\bar{v}'^2$, PDF(u), and PDF(v). Corresponding values of ϕ_i for the turbulent interwake region for these properties could be obtained directly from the correlations of Chen and Faeth.[4] The analogous values of ϕ_w for the laminarlike turbulent wake region for these properties were obtained as averages over the volume used to find f_w. Distributions of mean streamwise velocities contribute along with streamwise velocity fluctuations when the laminarlike turbulent wake contribution to $\bar{u}'^2$ is found because the arrival of particle wakes is random. The corresponding mean cross-stream velocity contribution to $\bar{v}'^2$ was ignored, however, because this velocity is small compared to cross-stream velocity fluctuations.[6] Similarly, the PDF of streamwise velocities within the laminarlike turbulent wake region accounted for both mean and fluctuating velocities; in contrast, the PDF of cross-stream velocities within the laminarlike turbulent wake region only considered velocity fluctuations because the mean cross-stream velocity contribution to this velocity component is small.[6] The details of these computations are described by Chen.[5]

Results and Discussion

Earlier measurements suggested that streamwise and cross-stream relative turbulence intensities were mainly functions of the dissipation factor, relatively independent of particle Reynolds number.[1–3] Present predictions supported this behavior as illustrated in Fig. 1. Measurements shown on the figure include those

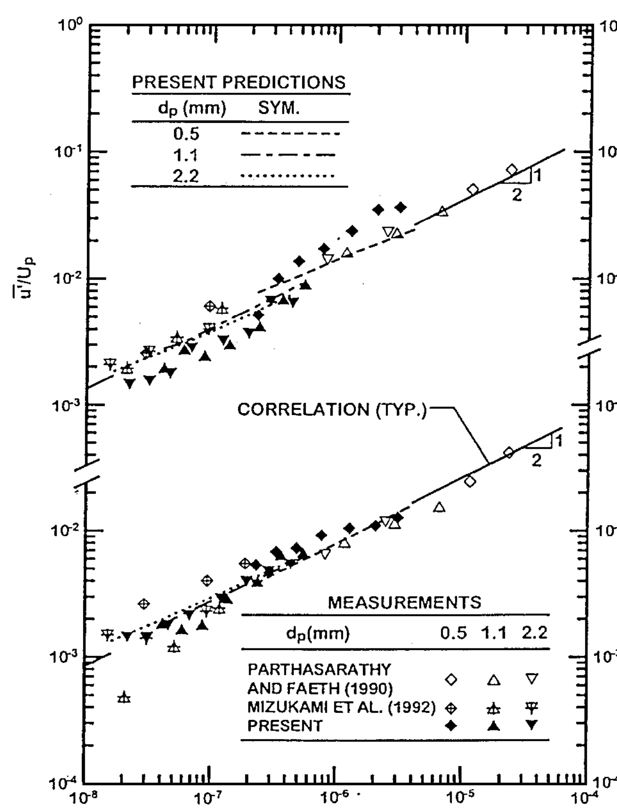

Fig. 1 Measurements and predictions of streamwise and cross-stream relative turbulence intensities of the overall flow as a function of dissipation factor: measurements of Parthasarathy and Faeth,[1] Mizukami et al.,[2] and Chen et al.[3]

of Parthasarathy and Faeth,[1] Mizukami et al.,[2] and Chen et al.[3] The corresponding predictions are limited to the last because of present limitations about the properties of laminarlike turbulent wakes.[6,7] The agreement between measurements and predictions is reasonably good. The predictions exhibit systematic effects of particle Reynolds number on relative turbulence intensities, but these effects are relatively small compared to experimental uncertainties over the available test range.[1–3] In addition, the measured variation of relative turbulence intensities as a function of the dissipation factor is seen to be predicted reasonably well.

The best-fit correlation of all the measurements illustrated in Fig. 1 yielded powers of D of 0.56 and 0.48 for the streamwise and cross-stream relative turbulence intensities with standard deviations of these powers of 0.03 and 0.02, respectively. As a result, there is no statistical significance between the powers of these fits and the power of $\frac{1}{2}$ suggested by the simplified theory of Parthasarathy and Faeth.[1] Thus, for consistency with the earlier work, the measurements were refitted by forcing the power to be $\frac{1}{2}$, yielding the correlations illustrated in Fig. 1 as follows:

$$\bar{u}'/U_p = 13.4D^{\frac{1}{2}}, \qquad \bar{v}'/U_p = 8.4D^{\frac{1}{2}} \tag{4}$$

where the standard deviations of the coefficients of these expressions are both 1.5 and the correlation coefficients of these fits are both 0.96, which is quite good. The anisotropy of the overall flow is significantly larger than the turbulent interwake region, e.g., 1.59 for the overall flow from Eq. (4) compared to 1.16 from the turbulent interwake region alone from Chen and Faeth.[4] This behavior follows as a result of the relatively large contributions of the mean velocities in the particle wakes to the streamwise flow disturbance, which is absent from the cross-stream flow disturbance. Notably, this effect is represented quite accurately by the predictions.

The enhanced flow disturbance of the streamwise velocity component compared to the cross-stream velocity component caused by the properties of the particle wakes is also observed in the PDFs of streamwise and cross-stream velocities. Typical examples for the 0.5- and 2.2-mm-diam particles are illustrated in Figs. 2 and 3. Measured and predicted streamwise and cross-stream velocity components for two particle fluxes for each particle size are illustrated,

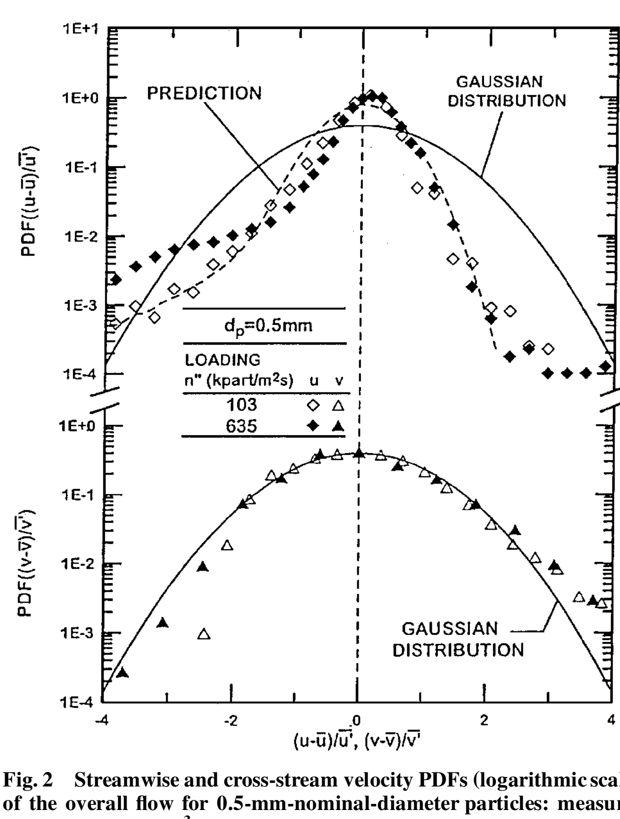

Fig. 2 Streamwise and cross-stream velocity PDFs (logarithmic scale) of the overall flow for 0.5-mm-nominal-diameter particles: measurements of Chen et al.[3]

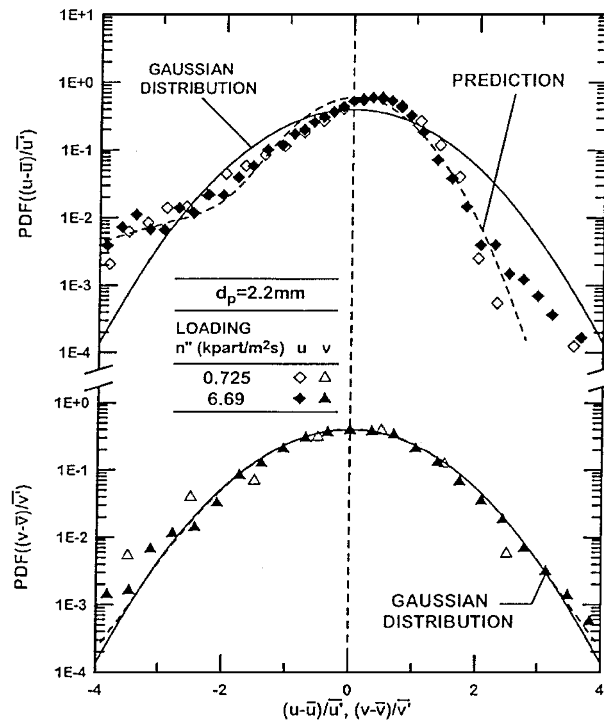

Fig. 3 Streamwise and cross-stream velocity PDFs (logarithmic scale) of the continuous phase for 2.2-mm-nominal-diameter particles: measurements of Chen et al.[3]

along with standard Gaussian distributions for reference purposes. The figures employ logarithmic ordinates so that effects of wake disturbances that dominate regions where the PDFs are small can be seen.

Both predictions and measurements in Figs. 2 and 3 show that the PDF of gas velocities in the cross-stream direction is nearly Gaussian, whereas the PDF of gas velocities in the streamwise direction is not well approximated by a Gaussian distribution even though both velocity components satisfy Gaussian PDFs in the turbulent interwake region. (See Ref. 4 for a discussion of PDFs within the turbulent interwake region.) This behavior can be explained by noting that the PDF(v) is caused entirely by turbulence in the turbulent interwake region and in the particle wake disturbance, and because both these contributions are represented quite well by Gaussian distributions it is not surprising that their combined effect is adequately fitted by a Gaussian distribution as well. Naturally, the turbulence contributions to the PDF of gas velocities in the streamwise direction behave in the same way; therefore, effects of mean velocities in wake disturbances are mainly responsible for the departure of the PDF of gas velocities in the streamwise direction from a Gaussian distribution. First of all, the mean velocities in the laminarlike turbulent wake disturbances only contribute negative velocities compared to the mean velocities of the flow, and these velocities generally are rather large negative velocities compared to turbulence levels in the turbulent interwake region.[3] Thus, these contributions are responsible for the upward bias, compared to the Gaussian distribution, seen at large negative velocities in Figs. 2 and 3. This upward bias is compensated by the behavior of the PDF of gas velocities in the streamwise direction at positive velocities where the maximum PDF of gas velocities in the streamwise direction shifts slightly to positive velocities while the PDF of gas velocities in the streamwise direction at large positive velocities is biased downward from the Gaussian distribution. The narrower (or more peaked) PDF of gas velocities in the streamwise direction near the mean velocity condition as the particle size (or Reynolds number) decreases, then follows from the reduced levels of turbulence in the wake disturbances. Such reduced disturbances concentrate the effects of wake disturbances to the mean velocity distributions, which implies reduced velocity variations and thus a more peaked PDF of gas velocities in the streamwise direction. This behavior is particularly promoted by laminarlike turbulent wake disturbances where streamwise turbulence intensities approach unity as a result of the unstable nature of wake flows.[6,7] Predictions and measurements also show that effects of particle fluxes on PDFs of u and v are small; this behavior generally agrees with numerous observations that the form of the PDFs of velocity fluctuations in turbulent flows is generally independent of the rate of dissipation of the turbulence. Finally, the agreement between measurements and predictions in Figs. 2 and 3 is reasonably good, with predictions correctly representing the effect of particle sizes and fluxes on the PDFs of u and v, particularly in the region where the square of the arguments of the PDFs are smaller than four and present experimental uncertainties are small. The main discrepancies between predictions and measurements are observed in regions where velocities are significantly larger or smaller than the most probable values. These regions correspond to rather small values of the PDFs of u and v, however, and the discrepancies can be largely attributed to both the small values of the PDFs and the sampling limitations of the present measurements (unfeasible orders of magnitude increases in testing times would be required to avoid this limitation).

Summary

In summary, the overall properties of the continuous-phase turbulence generated by uniform fluxes of monodisperse spherical particles moving at near terminal velocities were studied. It was found that many properties of these flows—including relative turbulence intensities, anisotropies, and PDFs—could be predicted reasonably well based on volume-averaged contributions of the conditional averages of these properties within the laminarlike wake disturbance and turbulent interwake regions of the flows. The predictions show that most of the unusual properties of these flows, compared to conventional homogeneous turbulent flows,[4–8] come about as a result of effects of mean velocities in the particle wakes, which cannot be separated from the turbulence (barring some type of wake discrimination system) because the arrival of particle wakes is random.

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Apparently First Closed-Form Solution for Frequency of Beam with Rotational Spring

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Nomenclature

A_j	=	functions defined in Eqs. (10–14)
a_j	=	inertial coefficients
b_I	=	stiffness coefficients
$D(\xi)$	=	flexural stiffness
G_1	=	coefficient defined in Eq. (19)
G_2	=	coefficient defined in Eq. (36)
k_1	=	rotational stiffness
L	=	length
m	=	order of polynomial in Eq. (2)
$R(\xi)$	=	inertial coefficient
$W(\xi)$	=	displacement
x	=	axial coordinate
α_j	=	coefficients defined in Eq. (3)
β	=	nondimensional elastic constant
ξ	=	x/L nondimensional axial coordinate
ω	=	natural frequency

Introduction

CLOSED-FORM solutions for natural frequencies of inhomogeneous beams with pinned supports were derived by Candan and Elishakoff.[1] This Note follows that study, to deal with beams supported by an elastic rotational spring, to derive apparently the first closed-form solution in the literature. To illustrate the feasibility of the proposed method, this Note considers the free vibrations of a beam with rotational elastic support at one end with the other end pinned.

Basic Equations

Consider inhomogeneous beams, governed by the differential equation

$$\frac{d^2}{d\xi^2}\left(D(\xi)\frac{d^2W}{d\xi^2}\right) - R(\xi)\omega^2 W(\xi) = 0 \tag{1}$$

Consider the inertial coefficient to be an mth-order polynomial function

$$R(\xi) = \sum_{i=0}^{m} a_i \xi^i \tag{2}$$

The beam's displacement is postulated as a fourth-order polynomial function:

$$W(\xi) = \alpha_0 + \alpha_1\xi + \alpha_2\xi^2 + \alpha_3\xi^3 + \xi^4 \tag{3}$$

The boundary conditions read

$$W(0) = 0, \qquad D(0)\frac{d^2W}{d\xi^2} = \left(\frac{k_1}{L}\right)\frac{dW}{d\xi}$$
$$W(1) = 0, \qquad D(1)W''(1) = 0 \tag{4}$$

which corresponds to the rotational spring at $\xi = 0$, of stiffness k_1. To have a compatibility of the order in the polynomial expressions,

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